

# AN ANALYSIS OF NONRECIPROCAL LIGHT COUPLER

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## ABSTRACT

A nonreciprocal light coupler without anisotropic crystal is analysed. An oblique magnetic field application causes nonreciprocal coupling between two dielectric waveguides through the magneto-optical medium between the guides. In principle, an optical isolator may be realizable which is tunable after fabrication.

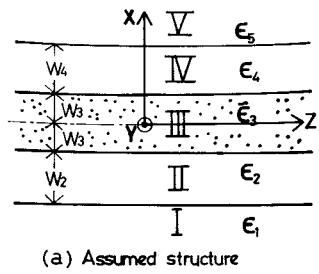
### Introduction

In future optical communication systems, nonreciprocal circuits especially thin film isolators will be strongly required, so that several authors have investigated such devices with various principles.<sup>1-6</sup> But, since they need a careful phase match for TE-TM mode coupling and/or an anisotropic crystal growth on magneto-optical crystal, it is difficult to realize them at present. Among all, they have a weak point that they are not tunable.

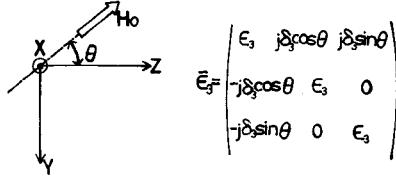
In the present paper we will analyze a nonreciprocal light coupler with isotropic material and magneto-optic crystal, which is able to control both the beat length and power transfer ratio by changing the direction of the applied magnetic field. This is applicable to an optical isolator which may be easily fabricated.

### Theory

We analyse the coupled modes in the structure shown in Fig.1(a). Each region I, II, IV, and V are



(a) Assumed structure

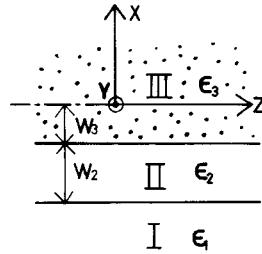


(b) The direction of the applied magnetic field

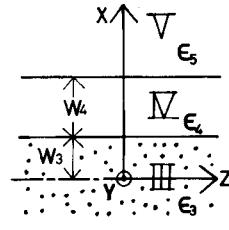
Fig.1 Geometry of the coupled waveguide system.

assumed lossless isotropic, and region III lossless magneto-optical with dielectric tensor  $\epsilon_3$ . Region II and IV are taken to be the waveguides. We call the structure shown in Fig.1 the perturbed system whose modes are assumed to be a little perturbed from the nearly degenerate  $TE_0$  and  $TM_0$  modes of system 1 and 2 shown in Fig.2, because of the mode coupling, and thus expressed by the linear combination of the latter<sup>2</sup>.

If the applied magnetic field is oriented to x, y, or z axis in Fig.1(a), the configuration is called P



Fundamental system 1



Fundamental system 2

Fig.2 Geometry of the fundamental system 1 and 2

(Polar), E(Equatorial), or L(Longitudinal) respectively. The E configuration does not bring about any coupling between TE and TM modes, but causes nonreciprocity only for TM mode. On the other hand, the L or P configurations couple the TE mode to the TM mode, but nonreciprocity does not occur. Therefore, we expect that superimposing the E configuration on the L or P configuration will make nonreciprocal mode coupling possible. We call these the PE or LE configuration and investigate the latter in detail.

Applying the variational method to the mode of the perturbed system<sup>2</sup>, we can obtain the following equations for the propagation constant of forward and backward waves in Fig.1(a), where we assume that the TE mode in the fundamental system 1 is nearly degenerate to the TM' in the fundamental system 2.

$$\beta^f = \beta_0 + \Delta\beta \pm Cf \quad (1)$$

$$\beta^b = \beta_0 - \Delta\beta \pm Cf \quad (2)$$

where

$$\beta_0 = (\beta^I + \beta^{II})/2 \quad (3)$$

$$\Delta\beta = \sin\theta \cdot \{ \omega\epsilon_0 \cdot \text{Re} [j\delta_3 \int_{-w_3}^{w_3} e_x^{II*} e_z^{II} dx] \} \quad (4)$$

$$C^f = \beta_0 \sqrt{\left( \delta - \frac{\Delta\beta}{\beta_0} \right)^2 + F^2} \quad (5)$$

$$C^b = \beta_0 \sqrt{\left( \delta + \frac{\Delta\beta}{\beta_0} \right)^2 + F^2} \quad (6)$$

and

$$\delta = (\beta^I - \beta^{II})/2\beta_0 \quad (7)$$

$$F = \frac{\omega\epsilon_0}{\beta_0} \left| \int_{-w_3}^{w_3} (-j\delta_3 \cos\theta) e_y^{I*} e_x^{II} dx \right| \quad (8)$$

In the equations above, superscripts I and II mean the fundamental system 1 and 2 respectively, and f, b the forward and backward waves in the perturbed system.

In addition,  $e_x$ ,  $e_y$ ,  $e_z$  are the  $x$ ,  $y$ ,  $z$  component of electric field in each system, and  $*$  means complex conjugate.

We can see the required nonreciprocity between forward and backward waves of the perturbed system in eqs.(1) and (2). Figure 3 shows the splitting of propagation constant  $\beta$ (a) for an ordinary reciprocal case and (b) for our nonreciprocal case. The power transfer between the fundamental system 1 and 2 is given as follows.

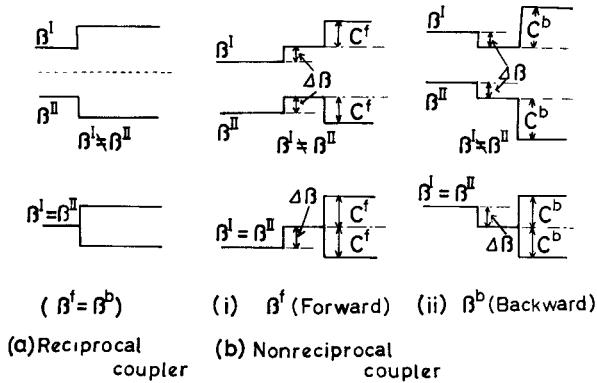


Fig.3 Splitting of propagation constant  $\beta$  for (a) reciprocal coupling and (b) nonreciprocal coupling in a nearly degenerate case ( $\beta^I \neq \beta^II$ ) and a degenerate case ( $\beta^I = \beta^II$ ).

#### Forward Wave

##### Power transfer ratio

$$\frac{P_1(z)}{P_1(0)} = 1 - \{1 - (\Delta_0^f)^2\} \sin^2 C^f z \quad (9)$$

$$\frac{P_2(z)}{P_1(0)} = 1 - \frac{P_1(z)}{P_1(0)} = \{1 - (\Delta_0^f)^2\} \sin^2 C^f z \quad (10)$$

Beat length

$$L^f = \pi / 2\beta_0 \sqrt{(\delta - \frac{\Delta\beta}{\beta_0})^2 + F^2} \quad (11)$$

#### Backward Wave

##### Power transfer ratio

$$\frac{P_1(z)}{P_1(0)} = 1 - \{1 - (\Delta_0^b)^2\} \sin^2 C^b z \quad (12)$$

$$\frac{P_2(z)}{P_1(0)} = 1 - \frac{P_1(z)}{P_1(0)} = \{1 - (\Delta_0^b)^2\} \sin^2 C^b z \quad (13)$$

Beat length

$$L^b = \pi / 2\beta_0 \sqrt{(\delta + \frac{\Delta\beta}{\beta_0})^2 + F^2} \quad (14)$$

where

$$\Delta_0^f = [\frac{\delta - (\Delta\beta/\beta_0)}{F}] / \sqrt{1 + [\frac{\delta - (\Delta\beta/\beta_0)}{F}]^2} \quad (15)$$

$$\Delta_0^b = [\frac{\delta + (\Delta\beta/\beta_0)}{F}] / \sqrt{1 + [\frac{\delta + (\Delta\beta/\beta_0)}{F}]^2} \quad (16)$$

$P_1(z)$ ; The power of TE mode in the fundamental system 1  
 $P_2(z)$ ; The power of TM mode in the fundamental system 2

Referring to eqs.(10),(13),(15) and (16), we can attain 100% power conversion if the conditions  $\beta^I - \beta^I = 2\Delta\beta$  for the backward wave are satisfied. On the other hand, in the ordinary reciprocal coupling, it occurs just at  $\beta^I = \beta^I$  regardless of the propagating direction. Furthermore, since  $\Delta\beta$  and  $F$  depend on the angle  $\theta$ , we can change  $\beta^I$  and  $\beta^I$  at which perfect conversion occurs.

#### Numerical Examples and Discussions

Numerical calculations are performed in the following way. At first we choose the width of the guide 2 arbitrarily, thus giving the value of  $\beta^I$ . After that, the direction of DC magnetic field and the width of the guide are determined so that  $\Delta\beta = 0$  and  $2nL^f = (2m-1)L^b$  are satisfied. If we here put  $n=m=1$ , we get  $2L^f = L^b$ , and hence, the power flow of a guide varies according to the propagation distance as shown in Fig.4(a). In this

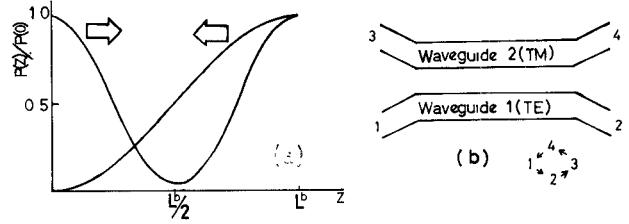


Fig.4(a) Power flow of each guide for forward and backward propagation.

(b) Resultant circulator operation.

case, the power in the guide 1 is taken out 100% at the another terminal for a forward propagation, while all the power in guide 1 is transferred to guide 2 for a backward propagation. One notices that it really behaves like a circulator as shown in Fig.4(b), if the input from port 3 or 4 is considered in a similar manner, so that one can realize an isolator terminating the proper two ports. We now choose YIG for a magneto-optical medium sandwiched between two isotropic guides, and 1.152  $\mu\text{m}$  for operating light wavelength.

The shorter the beat length is, the better. It is nearly inversely proportional to the overlap integral of TE and TM modes of guide 1 and 2 at YIG region as shown by eqs. (11) and (14). Since the overlap integral  $F$  is from the tails of the electric field out of each dielectric wave guide, the distance of two guides should be small to make the  $F$  value great. The  $F$  value becomes smaller, however, if the distance becomes too small, since the range of integration gets narrower.

Hence, the beat length has some minimum value if the guide distance is varied as shown in Fig.5.

Let the angle of DC magnetic field be varied and examine how the power at  $z=L^b$  changes. (Fig.6)

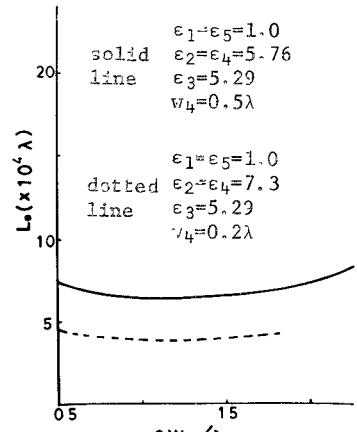


Fig.5 Beat length vs. wave guides distance. ( $\delta_3 = 3 \times 10^{-4}$ )

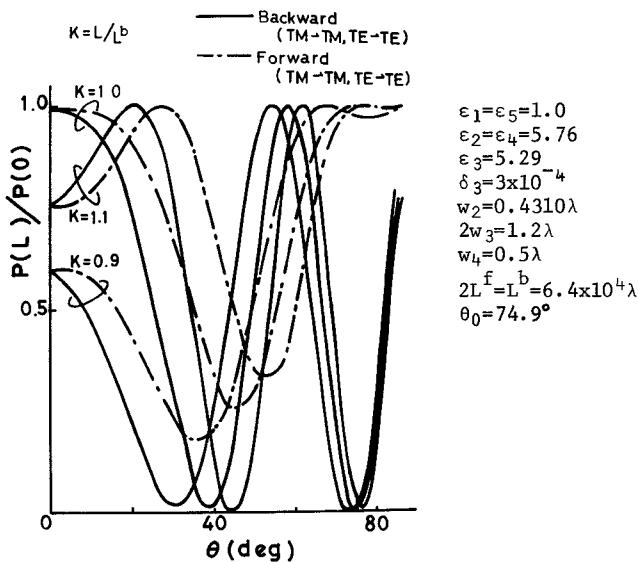


Fig. 6 Forward and backward wave power flow at  $L=0.9L^b$ ,  $L^b$  or  $1.1L^b$  as a function of magnetic field direction

Calculating with the same parameters as the solid line in Fig.5, we can see that at  $\theta=74.9^\circ$  the forward wave carries the full power while the contrary is the case for the backward wave. When the coupling length happens to be different from  $L^b$  we can see how the characteristics are degraded looking at the curves for  $L=1.1L^b$  or  $0.9L^b$ . Even in these cases we can recover the isolation characteristics by re-adjusting the DC field angle as shown in Fig.7.

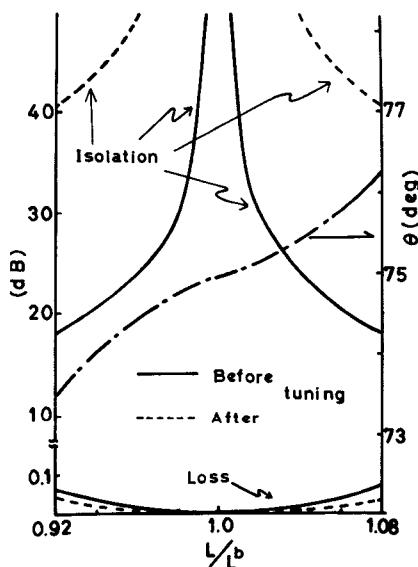


Fig. 7 Isolation and forward loss vs. coupling length deviation for 'before' and 'after' tuning. Tuning angle is also presented. (Parameters are the same as those in Fig.6)

The tuning capability stated above is also useful for other purposes. If we put  $\theta=90^\circ$  or  $58^\circ$ , the full backward wave energy is detected at the other terminal of the same guide. It will make an optical switch, considering that all the energy is transferred to another guide at  $\theta=74.9^\circ$ .

Going back to the beat length again, it is rather too large compared with other configurations previously investigated. In fact, the least value was about 4cm in our configuration as far as we use YIG for a magneto-optical medium. This is mainly because YIG is placed between two guides where rf electric fields are evanescent, and hence, we are investigating another configuration in which YIG itself is used as a waveguide.

#### Conclusion

We have proposed a nonreciprocal light coupler without anisotropic crystal. The device is able to control the power transfer and beat length by the direction change of the applied magnetic field, and is thus applicable to a tunable isolator, circulator or other devices.

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